Sums of Independent Random Variables

\( X \sim N(\mu, \sigma^2) \)

\( T = \sum_{i=1}^{n} X_i \)

\[ E(T) = E\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} E(X_i) = n\mu \]

\[ \text{var}(T) = \text{var}\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} \text{var}(X_i) = n\sigma^2 \]

\( T \sim N(n\mu, n\sigma^2) \)

Snowfall example

\( X \) is the snowfall in a day in inches, \( X \sim N(1, .5^2) \)

What is the probability of getting more than 18 inches of snow in a month?

What is the probability of getting between 6 and 12 inches of snow in a month?
Binomial Distribution

$X = 1$ if Heads, $0$ if Tails

$P(\text{Heads}) = p, P(\text{Tails}) = q$

$E(X) = np, \ Var(X) = pq$

Toss the coin $n$ times, and $r = \sum_{i=1}^{n} X_i$

$E(r) = E\left(\sum_{i=1}^{n} X_i\right) = nE(X_i) = np$

$\text{var}(r) = \text{var}\left(\sum_{i=1}^{n} X_i\right) = n \times \text{var}(X_i) = npq$

Polling example

$X = 1$ if for a measure, $0$ if against it

$P(X) = .65$

If you survey 100 people, what is the probability that 80 or more will vote for the measure? If you survey 1000 people?
Means of Independent Random Variables

\( X \sim N(\mu, \sigma^2) \)

\( \overline{X} = \frac{1}{n} \sum_{i=1}^{n} X_i \)

\[ E(\overline{X}) = E\left( \frac{1}{n} T \right) = \frac{1}{n} E(T) = \frac{1}{n} n \mu = \mu \]

\[ \text{var}(\overline{X}) = \text{var}\left( \frac{1}{n} T \right) = (\frac{1}{n})^2 \text{var}(T) = (\frac{1}{n})^2 n \sigma^2 = \sigma^2 / n \]

\( \overline{X} \sim N\left( \mu, \sigma^2 / n \right) \)

also known as the Sampling Distribution

SAT Example

X is the SAT scores of an individual, \( X \sim N(500, 100^2) \).

What is the probability that a random sample of 100 SAT takers has a mean, M, less than 400?

What is the probability that a random sample of 100 SAT takers has a mean, M, between 500 and 600?