Homework 2 (1/28/05)
DUE: Fri, Feb 4, beginning of class

The homework problems to be turned in next week are listed on this page (front), while the following pages contain practice problems to do on your own or with study groups. Doing more of the practice problems will further familiarize you with the necessary procedures and help you study for upcoming quizzes. The practice problems cover material that we’ve discussed and give a peek at some of the material we’ll be discussing next week. All of the problems need to be computed by hand (or with a calculator); please show all work. You may check your work on the computer (e.g., with Excel or SPSS).

Homework problems from the text:
- Aron & Aron, Chapter 6, Set 2: 17
- Aron & Aron, Chapter 7, Set 2: 14, 16, 20, 22

Additional homework problems:

A. A team of researchers is studying the effectiveness of polygraphs in detecting lies. The polygraphs measure a number of physiological responses, such as blood pressure, breathing rate, heart rate, and galvanic skin response (GSR, which measures sweat by seeing how easily electricity is conducted over the skin). Boris collected the heart rate data from 20 participants who were lying. He found that their mean heart rate was 76, with a standard deviation of 6. Natasha collected the blood pressure from 30 participants who were lying and found that their mean systolic blood pressure was 125, with a variance of 30. You conduct a third study, collecting heart rates of 30 individuals who are lying and find the following:

Data: 61, 66, 68, 70, 71, 72, 73, 73, 75, 75, 76, 77, 78, 78, 78, 79, 79, 80, 81, 82, 84, 85, 85, 85, 86, 86, 87, 87

1. Calculate the mean and variance for your sample. (20)
2. Compute the 90% confidence interval for the mean heart rate of lying individuals, using just your data. (40)
3. Pooling the data from your sample with the data from Boris’ sample, calculate the ‘best’ estimate, \( M \), for \( \mu \), the ‘best’ estimate, \( s^2 \), for \( \sigma^2 \), and the ‘best’ estimate, \( s \), for \( \sigma \). State the degrees of freedom of \( s^2 \). (30)
4. Compute the 99% confidence interval for the mean heart rate of lying individuals, using the pooled data. (40)
5. Test the null hypothesis (using a .05 significance level) that the systolic blood pressure of an lying individual is normally distributed with a mean of 120. (50)

B. The actual probability \( p \) of a teenaged girl being depressed nationally is 0.65. A study of 30 girls finds that \( r \) girls are depressed.

6. Find \( P(18 < r \leq 24) \). (30)
7. Find \( c \), such that \( P(r > c) = 0.15 \). (30)
8. Find \( P(r \leq 24 \mid r > 18) \). (30)

C. The following statement was made by a polling agency about a published poll that found that 54% of Americans believe in “love at first sight”.

The results below are based on telephone interviews with a randomly selected national sample of 1,000 adults. For results based on this sample, one can say with 95 percent confidence that the maximum error attributable to sampling and other random effects is plus or minus 3 percentage points.

9. Evaluate the statement above. Is the agency reporting on their survey practices accurately and honestly? Why or why not? (20)
10. The polling agency found that 540 (54%) believed in ‘love at first sight’ and 460 (46%) did not. Test the null hypothesis that 50% of the population believes in ‘love at first sight’. (40) [NOTE: There are multiple methods for solving this problem.]
Practice Problems (1/28/05)

Suggested practice problems from the text (with answers):

• Aron & Aron, Chapter 6, Set 1: 4, 6, 7, 9
• Aron & Aron, Chapter 7, Set 1: 1, 3, 5, 8, 9; Set 2: 22
• Study Guide, Chapter 6: 21-24
• Study Guide, Chapter 7: 21-23

A. Let \( p \) denote the unknown proportion of animals of a very large farm that are diseased. A random sample of animals is taken and each animal is examined for the disease.

1. If, in the sample of 100 animals, 33 were found to be diseased, calculate the 95% confidence interval for \( p \). [Ans. \( 0.33 \pm 0.1 = (.23, .43) \).]

2. Calculate how large a sample is needed if one wants to be 95% confident that \( p \) lies within .05 of the observed proportion of diseased animals in the sample. [Ans. \( n = 400 \).]

B. The level of hormones in healthy adults is Normally distributed with mean, \( \mu \), and variance, \( \sigma^2 \). In a sample of 36 healthy adults, the mean is 8.7 and the standard deviation is 0.72.

3. Calculate the 98% confidence interval for \( \mu \). [Ans. (8.42, 8.98).]

4. Calculate the 90% confidence interval for \( \sigma^2 \). [Ans. (.3643, .8076).]

5. A researcher wishes to use these data to test the hypothesis that \( \mu = 8.3 \). What should this researcher conclude? (Please state your answer briefly, but carefully.) [Ans. Since the 98% CI does not contain 8.3, 8.3 is not a plausible value for \( \mu \). So reject the researcher’s hypothesis.]

6. Calculate approximately how many healthy adults should be sampled if one wants to be 98% confident that \( \mu \) lies within 0.15 units of the observed sample mean. [Ans. \( n = 126 \).]

A. When the speed limit on a certain highway was 55 mph, the actual speed, \( X \), of a randomly selected driver was Normally distributed with a mean of 68.5, and a standard deviation of 7.0. (\( X \) may be regarded as a continuous random variable; i.e., you need not use a continuity correction in questions 1 and 2 below.)

1. Calculate the probability that a randomly selected driver would be driving at more than 65 mph. (20)

2. The Highway Patrol surveyed a random sample of 16 drivers, and computed the mean speed, \( \bar{X} \), for the sample. Calculate the probability that \( \bar{X} \) would be greater than 65 mph. (30)

B. The new speed limit is now 65 mph. The actual speed, \( X \), of drivers is still assumed to be Normally distributed under the new limit, and the Highway Patrol wants to obtain confidence intervals for the mean, \( \mu \), and the variance, \( \sigma^2 \), of this new distribution. To this end, the actual speeds of a random sample of 25 drivers were noted. The sample mean was \( \bar{X} = 68.7 \), and the sample standard deviation was \( s = 6.0 \).

3. Calculate the 90% confidence interval for \( \mu \). (40)

4. Calculate the 90% confidence interval for \( \sigma^2 \). (40)

5. Suppose the Highway Patrol wants to have an approximate 95% confidence interval for \( \mu \) that has a half-width of only 2 mph. Calculate how large a sample they should observe. (40)

6. Let \( p \) (0 < \( p \) < 1) denote the proportion of drivers driving above the new speed limit. To estimate \( p \), the Highway Patrol observes \( n \) drivers, noting whether or not each drives above the new limit. In order to obtain an approximate 95% confidence interval for \( p \) that has a half-width of only 0.03, calculate how large \( n \) should be. (40)
B. A quiz consists of 64 multiple-choice questions. Each question has 4 alternatives, one of which is correct. A student guesses on each question. Let \( r \) be the number of questions the student gets right.

6. Calculate the probability, \( P(14 < r \leq 20) \). (30) (Ans. \( \mu = np = 64/4 = 16 \). \( \sigma = \sqrt{npq} = 3.46 \). Using the continuity correction, we compute \( P(14.5 < r < 20.5) = ... = P(-.434 < Z < 1.30) = .5696 \).)

7. Calculate \( c \), such that \( P(r > c) \approx 0.025 \). (30) (Ans. \( P(r > c-0.5) = ... c = 23.28 \approx 23 \).)

8. Another student gets 28 questions correct on the quiz. Please state a brief, cogent reason why this student’s performance is or is not significantly better than chance performance. (20) (Ans. From #7, we know that 23 is a ‘large’ score if a student guesses. Thus, 28 is unlikely if the student guesses; 28 is significantly better than chance performance and we would conclude that the student knows the material.)

C. Let \( p \) be the proportion of SU students who support the strike. A random sample of students is selected and asked about their support for the strike.

9. If, in a sample of 500 voters, 340 support the strike, calculate the approximate 95% confidence interval for \( p \). (30) (Ans. (.635, .725).)

10. Suppose that you wish to have a margin of error of 3% (i.e., 0.03) in estimating \( p \). Calculate the size of the sample of students you would need to survey. (30) (Ans. \( n = 1111 \).)

D. An aspiring professional athlete is asked to run the 40 yard dash many times (with sufficient rest in between each run). The owner of a team wishes to use the observed times \( X \) for the run in order to estimate (a) the athlete’s true mean time, \( \mu \), to run this distance, and (b) the variance, \( \sigma^2 \), of the distribution of \( X \). The athlete provides \( n = 9 \) times, \( x_i \). The mean of the sample is \( \bar{X} = 4.45 \) seconds, and the sample standard deviation is \( s = 0.087 \). It may be assumed that the time, \( X \), in a dash is Normally distributed.

11. Calculate the 90% confidence interval for \( \mu \). (30) (Ans. (4.396, 4.504).)

12. Calculate the 90% confidence interval for \( \sigma^2 \). (40) (Ans. (.0039, .0223).)

A. Rodney, the researcher, found that, in a random sample of 49 California mothers, the average ‘length of pregnancy’ was 36.2 weeks, and the standard deviation of the sample was 1.4 weeks.

1. Test the null hypothesis that the population average (\( \mu \)) for California pregnancies is 37 weeks (which, if you are interested, is average length of pregnancies for the whole U.S.A.). (Ans. Use the t-test as the ‘best’ choice; however, since \( n \) is ‘large’, it is okay to use the z-test.)

2. Compute the 90% confidence interval for the population variance (\( \sigma^2 \)). (Ans. Use the chi-square distrn. with 48 df.)

C. Rejection Regions, \( R \). Suppose that, in order to test the null hypothesis (\( H_0 \)), “\( \mu = 10 \)”, it is appropriate to use \( t \) with 6 degrees of freedom. For each choice of significance level, \( \alpha \), and choice of the alternative hypothesis, (\( H_1 \)), please state the rejection region, \( R \), in terms of \( t \) (e.g., as “\( R: t < 2 \)”).

9. State \( R \) when \( \alpha = .05 \) and \( H_j \) is “\( \mu \neq 10 \)”.

10. State \( R \) when \( \alpha = .05 \) and \( H_j \) is “\( \mu < 10 \)”.

11. State \( R \) when \( \alpha = .10 \) and \( H_j \) is “\( \mu > 10 \)”.

(Ans. R: \( t < -2.447 \) OR \( t > 2.447 \).)

(Ans. R: \( t < -1.943 \).)

(Ans. R: \( t > 1.440 \).)